Slow phase relaxation as a route to quantum computing beyond the quantum chaos border

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We reveal that phase memory can be much longer than energy relaxation in systems with exponentially large dimensions of Hilbert space; this finding is documented by 50 years of nuclear experiments, though the information is somewhat hidden. For quantum computers Hilbert spaces of dimension 2^{100} or larger will be typical and therefore this effect may contribute significantly to reduce the problems of scaling of quantum computers to a useful number of qubits.

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To solve problems intractable up to now, quantum computers (QC) should operate with $n>100$ interacting qubits. Georgeot and Shepelyansky (GS) considered a two-body random Hamiltonian as a generic model for QC hardware, and performed numerical simulations for $n\leqslant 15$ [1]. They claim that information loss, referred to as meltdown of the QC, occurs on a time scale given by qubit mixing of eigenstates. Since the dimension of Hilbert space grows exponentially and the spectral span only linearly, this poses stringent conditions on the interactions among qubits. However, to test these restrictions for realistic n, we need, alas, a working QC. Instead, we resort to old and new nuclear data. We find that using proton inelastic scattering on heavy nuclei as a quantum protocol, the eigenstate mixing time is orders of magnitude shorter than that required for information loss. Thus, in exponentially large Hilbert spaces, phase memory, not usually considered, is greatly enhanced.

This paper is addressed to readers without specific nuclear physics background. Therefore, we only briefly outline a physical picture of the phenomenon which reveals that heavy nuclei provide a seed for a scaling of QC. More detailed consideration of nuclear specific aspects of the problem will be presented in an extended version of this paper.

The feasibility of quantum computing on a large scale has been studied from different viewpoints. The most common approach is a time dependent one, related directly to the increase of errors as a function of the number of gates and qubits [2,3]. Fidelity or more specific process-related benchmarks are used to get a reliable picture. This approach is self-defeating if one wants to scale it to a QC of useful size, and simultaneously go beyond perturbation theory [4]. A functioning QC would be needed to make the correct calculation with which the perturbed one is compared.

GS point out that, for chaotic dynamics, the identity of functions on individual qubits may be lost at a rate faster than the quantum protocols [4]. This so-called meltdown of the QC would put very serious limitations on its implementation. This analysis is based on standard theory of relaxation in quantum many-body systems.

The basic assumptions involved are the following: A qubit is normally a two-level system, with an average energy difference Δ_0. For n qubits the level density grows exponentially with n. This, according to GS, imposes stringent restrictions on the strength and/or form of the interaction among qubits, since otherwise many noninteracting n-qubit states $|\Psi_i\rangle$ will be mixed and the QC melts down. These limitations are particularly damaging since chaotic dynamics can stabilize quantum computation against external errors [5,6].

To investigate parameter values for which the QC can indeed operate, GS analyzed the statistical properties of the two-body random Hamiltonian $H=\Sigma_i L_i\sigma_i^z+\Sigma_{i<j}J_{ij}\sigma_i^x\sigma_j^x$, where σ_i are the Pauli matrices for qubit i. The random numbers L_i and J_{ij} are distributed, respectively, in the intervals $[\Delta_0-\delta/2,\Delta_0+\delta/2]$ and $[-J,J]$. The analysis [1] assumes nearest neighbor coupling.

In the noninteracting qubit basis the eigenfunctions $|\phi\rangle$ are obtained. For $n=12$, $W_i=|\langle\Psi_i|\phi\rangle|^2$ is plotted as a function of the noninteracting multiqubit energy E_i for two values of J/Δ_0. For $J/\Delta_0=0.02$, W_i is very narrowly distributed, whereas for $J/\Delta_0=0.48$ the computer eigenstates become a broad and somewhat random mixture of the quantum register states $|\Psi_i\rangle$. In the drastic language of GS the meltdown has occurred before the quantum protocol could be realized. This implies a time scale, which will be introduced below using the standard language of statistical nuclear physics.

Wigner, some 50 years ago [7], introduced the spreading width $\Gamma^{\downarrow}$, in the context of many-body problems consisting of n interacting particles, with large but finite n. $\Gamma^{\downarrow}$ indicates the spread of W_i, and $\hbar/\Gamma^{\downarrow}$ is the energy relaxation time for which, according to standard theory, all memory of the initial state is lost. We shall return to this interpretation later. Interchanging the roles of the eigenbasis and the single-particle basis, the local density of states (LDOS) is obtained. Its width is typically again $\Gamma^{\downarrow}$.

Unfortunately, for large n it is impossible to perform the calculations of GS since the dimension $N_H=2^n$ of the Hilbert space grows exponentially. We therefore propose a different approach, using experimental data involving heavy nuclei. The nucleus is an ideal laboratory to study many-body systems, since nuclear interactions are so strong that external perturbations can be neglected.

Consider some scattering process, such as inelastic proton-nucleus scattering to be the quantum protocol. The single-particle basis is the quantum register, the entrance

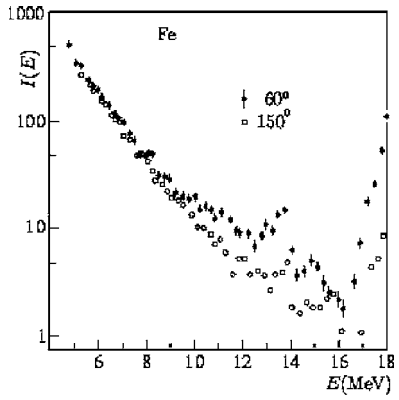

FIG. 1. Scaled proton spectra $I(E)$ (in arbitrary units) at forward and backward angles for 18 MeV proton inelastic scattering on iron (reproduced from Fig. 3 of Ref. [8]). They represent relative LDOS of the residual nucleus for high excitation energy, i.e., low proton energy.

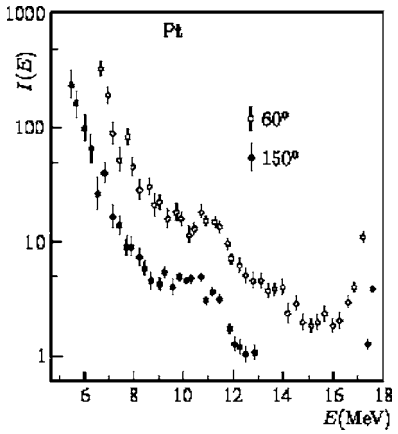

FIG. 2. Similar spectra as in Fig. 1 for a platinum target (reproduced from Fig. 9 of Ref. [8]). Note that the vertical scale no longer represents LDOS in any range because of the significant difference between backward and forward angles.

channel represents the loading process, and the output is the readout. The question is the following: How long is the memory and is it given by the spreading width or, equivalently, by the width of LDOS? Since experiments of this type have been available for 50 years [8] and are still performed [9], this question can be answered, the nucleus playing the role of the QC. We address here the phase memory of the process, which is not usually considered in the field of compound nuclear reactions, because energy relaxation was at the center of attention. Phase relaxation is usually assumed to be at least as fast or faster than energy relaxation, in particular because random phase approximations are so successful in diverse aspects of nuclear physics.

We revisit the 1954 paper of Gugelot [8] describing the inelastic scattering of 18 MeV protons off several targets, including light nuclei such as aluminum, medium heavy ones, for example, iron, nickel, copper, silver, and tin, as well as heavy nuclei such as platinum and gold. The energy spectra of the outgoing protons are detected at different angles. The raw data are scaled with the proton energy E times the penetration factor of the Coulomb barrier to produce $I(E)$. At proton energies well below this barrier, where compound reactions dominate, the scaled spectra should represent LDOS of the residual nucleus and, therefore, be angle independent. This happens for light and medium nuclei, as exemplified in Fig. 1 for iron. Surprisingly this is not so for heavy nuclei, as shown in Fig. 2 for platinum. The curves are different, but the exponential slope at low energies is the same for both angles, indicating that energy relaxation has occurred at ≈ 0.7 MeV per proton. Gugelot stresses that there are no spurious experimental effects in the platinum data, and that gold spectra look similar.

The reader might wonder whether state of the art nuclear theories do not contradict our assumption that energy is equilibrated. The evolution of a nuclear reaction is supposed to proceed via a series of two-body nucleon-nucleon collisions, which successively form states of increasing complexity. On each stage of the reaction a distinction is made between continuum states and quasibound states. Emissions from the continuum states result in multistep direct reactions [10–12], and decay of the quasibound states results in multistep compound processes [10,13]. The compound nucleus is formed at the last, most complex configuration of the chain of the quasibound states. The multistep direct reactions originate from the decay of the simplest configurations of the chain resulting in forward-peaked angular distributions. The multistep compound reactions give rise to angular distributions symmetric about 90 degrees.

We have used the exciton model [14] to evaluate relative contributions of multistep direct, multistep compound, and compound nucleus processes for the p+Pt inelastic scattering for the proton outgoing energy of 7 MeV. Fitting the entire energy range for forward angles we found that the compound nucleus cross section constitutes 90%, while multistep direct and multistep compound are about 5% each.

In Fig. 3 more recent proton angular distributions obtained from scattering data of neutrons [9] and protons [15] on a bismuth target, confirm the forward peaking. Calculations of the multistep direct reaction contribution for the inelastic proton scattering show that for the outgoing proton energy of 10 MeV or less this contribution is negligible (see

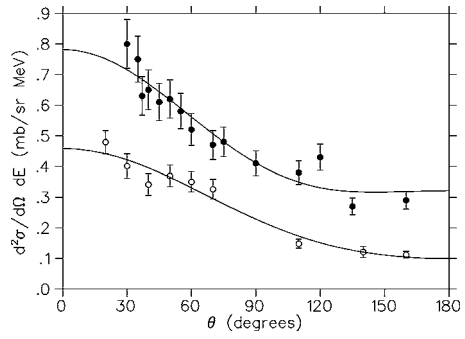

FIG. 3. Angular distribution (dots) of inelastic sub-Coulomb 9 MeV protons measured with a 62 MeV beam on a bismuth target [15]. Circles represent a similar distribution of 9 ± 1 MeV protons resulting from 62.7 ± 2 MeV neutron induced reactions on bismuth [9]. The full lines are fits with Legendre polynomials up to second order.

Fig. 10 in Ref. [16]) and that the angle integrated cross section around 9 MeV proton outgoing energy is almost entirely given by the compound nucleus evaporation (see Fig. 17 in Ref. [14]). Yet memory of the direction of the incident beam is clearly retained.

The essential question is the following: How much time did the protocol, i.e., the reaction process, take as compared to the energy relaxation time $\hbar/\Gamma^{\downarrow}$. Using standard nuclear physics estimates [17], $\Gamma^{\downarrow}$ for platinum is of the order of 1 MeV. Assuming that we are in a compound state, we can estimate the total decay width $\Gamma^{\uparrow} \approx 0.02$ keV (see Fig. 7 in Ref. [18]). This leads to a process time five orders of magnitude longer than $\hbar/\Gamma^{\downarrow}$. The theoretical estimates given for both widths should not be off by more than a factor of 3 leaving at worst still four orders of magnitude between the two time scales. We therefore clearly see that there is strong old and new experimental evidence, that $\hbar/\Gamma^{\downarrow}$ is not the time scale for information loss.

Estimates of the effective dimension of Hilbert space, $\Gamma^{\downarrow}/D$, can be obtained from the spreading width [17] and the density of states of the compound nucleus D [19]. These dimensions and the number of qubits needed to roughly equate them, are 10^{20} ($\approx$67 qubits) for p+Pt and 10^{9} ($\approx$30 qubits) for p+Fe. In view of such dimensions, digital computations to confirm this effect cannot be performed. We can speculatively ask how far nuclear physics can take us in the oposite direction? Superheavy nuclei with masses up to 277 have been detected. Assuming similar excitation energies this would bring us to $\approx$90 qubits. While we may expect even longer phase relaxation times, the difficulties of experiments on asymmetries in angular distributions of evaporated nucleons seem quite formidable.

In nuclear physics one may think of two possible theoretical explanations of this phase memory persistence. There are indications that random two-body interactions in exponentially large Hilbert spaces need not lead to chaotic states even if all pairs interact [20]. We then could have large spreading widths, i.e., strong interaction, but fairly small participation ratios of the expansion of one basis in terms of the other, as expected for systems with Poissonian statistics in the strong-coupling case [21]. This would imply that states are not evenly populated, and the average proton energy of 0.7 MeV is then not easily explained. We therefore prefer to assume that the time scale for phase relaxation is much longer than that for energy relaxation. One of us has proposed such ideas some time ago [22,23] showing that very weak correlations between different angular momenta may be considerably enhanced in exponentially large Hilbert spaces, even if thermalization occurs for each angular momentum. This theory predicts that odd terms in a Legendre expansion of the angular distribution will not vanish, but be determined by the ratio of decay time and phase relaxation time [24]. The corresponding fits are shown in Fig. 3 and the two time scales agree. This is consistent with our statement that phase relaxation is orders of magnitude slower than energy relaxation.

The good news is then that there are 50 years of strong experimental evidence that the energy relaxation time is not the relevant time scale that limits memory conservation in a system of many qubits. We have identified an effect, observable only in exponentially large Hilbert spaces, that introduces a much longer time scale for phase memory in a many-body system. In principle, this effect allows scaling to a large number of qubits, although it will certainly not replace stabilization techniques developed for small-n QC [2–6]. The bad news is that we need more theoretical insight for the appropriate engineering of a QC to take advantage of this effect.

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